Solutions to JEE Advanced Home Practice Test -2 |JEE 2024 | Paper-1

PHYSICS

NUMERIC TYPE

1.(8) $R = \frac{d}{2}$

$$\frac{GM^2}{d^2} = M\left(\frac{2\pi}{T_S}\right)^2\left(\frac{d}{2}\right)$$

$$T_S = 2\pi\sqrt{\frac{d^3}{2GM}}$$

$$F = \frac{2GMmZ}{(R^2+Z^2)^{3/2}} = \frac{2GMmZ}{R^3} = \left(\frac{16GMm}{d^3}\right)Z$$

$$\left(\frac{d}{2} = R >>> Z\right)$$

$$T_P = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{d^3}{16GM}} == \frac{\pi}{2}\sqrt{\frac{d^3}{GM}} \;;\; \frac{T_S}{T_P} = 2\sqrt{2} \;;\; \left(\frac{T_S}{T_P}\right)^2 = 8$$

2.(2.20) $E = \sigma T^4$

$$\Rightarrow \frac{E_2}{E_1} = \frac{T_2^4}{T_1^4} = 256 = 4^4 \Rightarrow T_2 = 4T$$

Also, $\lambda_1 T_1 = \lambda_2 T_2$ gives $\lambda_2 = \frac{\lambda_1}{4} = 1000\mathring{A}$

$$KE_{\max} = \frac{hc}{\lambda_2} - \phi$$

$$\Rightarrow \frac{12400}{1000} - \phi = 12.4 - \phi \qquad \text{..........(1)}$$

$$\Delta E_{2\to4} = 13.6\times4\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = 13.6\times4\times\frac{3}{16} = 10.2\ eV \qquad \text{..........(2)}$$

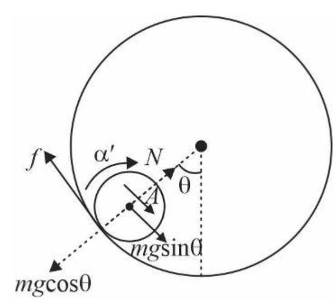
From (1) and (2) $12.4 - \phi = 10.2$

$\Rightarrow \phi = 2.2\ eV$

3.(6.25) $mg\sin\theta - f = mA$ … (1)

$fr = \frac{2}{5}mr^2\alpha^1$ … (2)

$A - r\alpha^1 = -R\alpha$ … (3)



$$f = \frac{2}{5}mr\alpha^1 = \frac{2}{5}m(A + R\alpha) = \frac{2}{5}mR\alpha \;\; [A = 0]$$

$$mg\sin\theta - f = mA \qquad \Rightarrow \qquad mg\sin\theta = f$$

$$\frac{mg}{2} = \frac{2}{5}mR\alpha$$

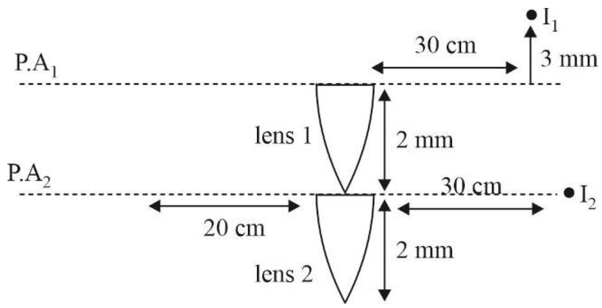
$$\alpha = \frac{5g}{4R} = \frac{50}{8} = 6.25 \text{ rad/s}^2$$

4.(5) For both lens

$$\frac{1}{V}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{V}-\frac{1}{(-20)}=\frac{1}{12}$$

$$V=30\,cm$$



For lens 1

$$\frac{h_i}{h_0}=\frac{V}{u} \Rightarrow h_i=-2\times\frac{30}{-20}=3\,mm$$

Distance between I_1 & I_2 is 5 mm

5.(2.40) $F(4l_0)=\frac{1}{2}k(2l_0)^2$ and $mg=k(2l_0)\left(\frac{3}{5}\right)$ $\therefore\ F=\frac{5}{12}mg$

6.(9) For T_1 $m\omega_1^2 x=9Kx$ or $\omega_1=\sqrt{\frac{9k}{m}}$

$$T_1=2\pi\sqrt{\frac{m}{9k}}$$

For T_2 $m\omega_2^2 x=Kx$ or $\omega_2=\sqrt{\frac{k}{m}}$

$$T_2=2\pi\sqrt{\frac{m}{k}}$$

7.(6) Step 1 : S_1 closed $\left(C_{eq}=\frac{C}{2}\right)$

$$\omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2\times10^{-2}\times\frac{1}{2\pi^2}\times10^{-2}}}=100\pi\ rad/s$$

Also, $q=q_{\max}\cos\omega t$

$$i=-q_{\max}\omega\sin\omega t$$

$$t=\frac{1}{400}$$

$$q=4\sqrt{3}\cos\frac{100\pi}{400}=2\sqrt{6}\mu C$$

$$i=-4\sqrt{3}\times100\pi\sin\frac{100\pi}{400}=-200\sqrt{6}\pi\,\mu A$$

Step 2 : S_2 closed $(C_{eq}=C)$

For max charge on capacitor (Q_0)

$$\frac{1}{2}i^2+\frac{1}{2}\frac{q^2}{C}=0+\frac{Q_0^2}{2C};\ \frac{2\times10^{-2}}{2}\times\left(200\sqrt{6}\pi\right)^2+\frac{\pi^2}{2\times10^{-2}}\left(2\sqrt{6}\right)^2=\frac{\pi^2Q_0^2}{2\times10^{-2}}$$

$$2400+1200=\frac{100}{2}Q_0^2$$

$$Q_0^2=36\times2\ ;\ \ Q_0=6\sqrt{2}\,\mu C$$

8.(2.50) $C = \epsilon_0 A/d$

When 4 and 5 are not connected

$$C_{eff} = \frac{5C}{3}$$

Charge given by battery $q_1 = C_{\text{eff}} \cdot V = \frac{5CV}{3}$

When 4 and 5 are connected

$$C_{eq} = \frac{5}{2}C; \qquad q_2 = \frac{5}{2}CV$$

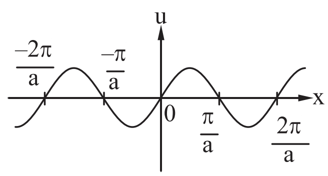
work done by battery $= V\left[\frac{5}{2}CV - \frac{5}{3}CV\right]$

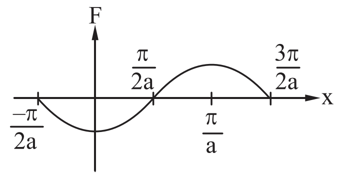
$$= \frac{5CV}{6} \times V = \frac{5}{6}CV^2 = \frac{5}{6} \times 30 \times 100 = 2500\mu\text{J} = 2.50 \text{ mJ}$$

MULTIPLE CHOICE

9.(BC) $u = u_0 \sin ax$

$$F = -\frac{du}{dx} = -u_0 a \cos ax$$





$\Rightarrow$ If particle is released from rest at origin it will oscillate between $x = 0$ and $x = -\frac{\pi}{a}$.

$\Rightarrow$ Minimum K.E at origin to cross the hill is.

$$\frac{1}{2}mu_{\min}^2 = u_0; \qquad v_{\min} = \sqrt{\frac{2u_0}{m}}$$

If $v > v_{\min}$ it will continue to travel to infinity.

$\Rightarrow$ $x = \frac{\pi}{2a}$ is a position of unstable equilibrium

So particle will not perform SHM about this position

10.(ABCD) $i_2 = \frac{24}{12} = 2A$ (ohm's law)

At B (KCL)

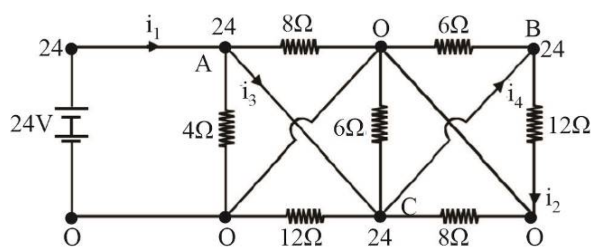
$$i_4 = i_2 + \frac{24}{6} = 2 + 4 = 6A$$

At C (KCL)

$$i_3 = i_4 + \frac{24}{6} + \frac{24}{8} + \frac{24}{12} = 15A$$

At A (KCL)

$$i_1 = i_3 + \frac{24}{8} + \frac{24}{4} = 24A$$



11.(ABD) At equilibrium

$F_{net} = 0$ (on piston)

$P_2 S = Kx_0 \Rightarrow P_2 = \dfrac{Kx_0}{5}$ (P_2 : final pressure of gas)

Also, process is adiabatic

$$w_{gas} = \frac{P_2V_2 - P_1V_1}{1-V}$$

$$= \frac{\frac{kx_0}{S}(V_0 + x_0 S) - P_0V_0}{1-\frac{7}{5}} = \frac{-5}{2}\left(\frac{kx_0v_0}{5} + kx_0^2 - P_0V_0\right)$$

$\Delta U = -w$ (adiabatic process)

$$= \frac{5}{2}\left(\frac{kx_0v_0}{5} + kx_0^2 - P_0V_0\right)$$

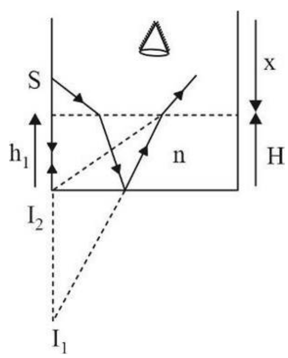
Further,

$w_{gas} + w_{ext} + w_{spring} = \Delta KE$ (work KE theorem on piston)

$$w_{ext} = -w_{gas} - w_{spring} = \frac{5}{2}\left(\frac{kx_0v_0}{5} + kx_0^2 - P_0V_0\right) + \frac{kx_0^2}{2} = \frac{5V_0}{2}\left(\frac{kx_0}{5} - P_0\right) + 3kx_0^2$$

12.(AB) Speed of liquid rise $= \dfrac{h}{T}$

Case-1 :



Position of I_1 : $n(h_1 - H) + H$ (below mirror)

Position of final image (I_2) : $(h_1 - H) + \dfrac{2H}{n} + x$ (from observer)

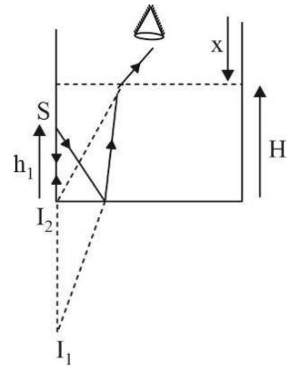
$$\text{Speed of image} = \frac{-dH}{dt} + \frac{2}{n}\frac{dH}{dt} + \frac{dx}{dt}$$

$$\left(\frac{dI_2}{dt}\right) = \frac{2}{n}\frac{dH}{dT} - \frac{2dH}{dT}$$

$$= \left(\frac{1}{n} - 1\right)\frac{2dH}{dt} = \frac{2h}{T}\left(\frac{1}{n} - 1\right)$$

$= \dfrac{-2h}{T}\left(1-\dfrac{1}{n}\right)$ (negative indicates towards observer)

Similarly Case- II



Position $(I_1) = h_1$ below mirror

Position $(I_2) = \dfrac{(h_1 + H)}{n} + x$ (from observer)

Differentiate w.r.t. time

Speed $(I_2) = \dfrac{1}{n}\dfrac{dH}{dt} + \dfrac{dx}{dt} = \left(\dfrac{1}{n} - 1\right)\dfrac{h}{T}$

Speed of image $= \dfrac{-h}{T}\left(1-\dfrac{1}{n}\right)$ (negative indicate towards the object)

13.(ABD)

As at $C, |\vec{B}| = 0$ also as at $x = l$ and just to the right of B, $|\vec{B}| = -\infty$.

$\therefore$ current in A is along z-axis and in B along negative z-axis

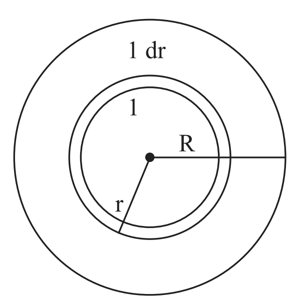
as $|\vec{B}|$ at a general distance x from A is $B = \dfrac{\mu_0}{4\pi} 2I\left(\dfrac{2}{x} - \dfrac{1}{x-l}\right)$

from this we get $x_1 = 2l$ also at $x = x_2, \dfrac{dB}{dx} = 0$ from this $x_2 = \left(\sqrt{2}+2\right)l$

14.(AC) Velocity of flow $v = v_0\left(1-\dfrac{r^2}{R^2}\right)$.

Mass of the liquid in cylindrical section of radius r and thickness dr is $dm = 2\pi r dr \cdot L\rho$.

Kinetic energy, $dK = \dfrac{1}{2} dm v^2$



$= \dfrac{1}{2} 2\pi L\rho r dr\, v_0^2\left(1-\dfrac{r^2}{R^2}\right)^2 \quad \therefore \quad K = \pi L\rho v_0^2 \int_0^R \left(1-\dfrac{r^2}{R^2}\right)^2 r dr$

(i.e.,) $K = \dfrac{\pi}{6} L\rho v_0^2 R^2$

The viscous drag exerts a force on the tube, given by,

$F = -\eta A\left(\dfrac{dv}{dr}\right)_{r=R} = 4\pi\eta L v_0$

Pressure difference $\Delta P = \dfrac{\text{Force}}{\text{Area of cross-section}} = \dfrac{4\pi\eta L v_0}{\pi R^2} = \dfrac{4\eta L v_0}{R^2}$

MATRIX MATCH

15.(C) (I) F = 0 ; closed loop in uniform B

$\tau = 0$; $\vec{M}$ & $\vec{B}$ are not aligned

(II) F = 0 ; closed loop in uniform B

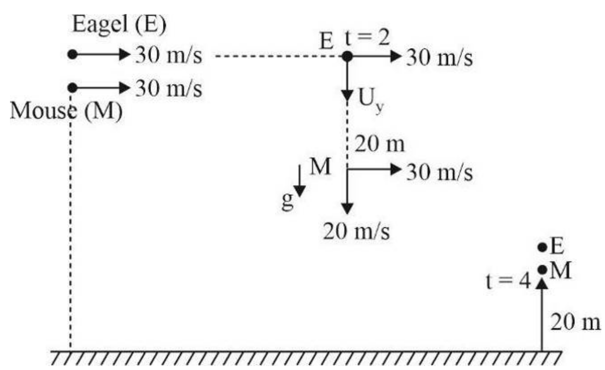
$\tau = 0$; $\vec{M}$ & $\vec{B}$ are not aligned

(III) $F \neq 0$; not closed loop

(IV) $F \neq 0$; closed loop in non uniform B

$\tau = 0$; force on each element is in radial direction

16.(B)



Time between dive and recapture

$-60 = -20t - \frac{1}{2}gt^2$ (for mouse)

$t^2 + 4t - 12 = 0$

$t = 2\,\text{sec}$

For eagle : $80 = U_y t \Rightarrow U_y = \frac{80}{2} = 40\,m/s$

Eagle diving speed $= \sqrt{30^2 + 40^2} = 50\,m/s$

Diving distance $= 50 \times 2 = 100\,m$

Diving angle from vertical $= \tan^{-1}\frac{3}{4} = 37°$

17.(A) (I) $(P/T)^{5/2} = C$ (Put $P = \frac{nRT}{V}$)

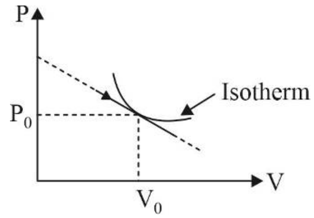
$VT^{3/2} = C \Rightarrow$ thus, increase in V decrease T

$P/T^{5/2} = C$ (Put $T = \frac{PV}{nR}$)

$PV^{5/3} = C \text{ or } PV^{\gamma} = C$ (adiabatic process)

Thus, $Q = 0$

(II) Slope of line graph is $\frac{-P_0}{V_0}$ which is same as slope of isotherm at P_0, V_0. Thus the line is tangent.



Thus, temperature increases as straight line graph approaches V_0.

$Q = \Delta U + w$

ΔU is positive and w is also positive (as gas is expanding). Hence heat is absorbed by the gas.

(III) Adiabatic compression, temperature increases

(IV) $\rho V = C$

$\left(\frac{m}{V}\right)\left(\frac{f}{2}nRT\right) = C$

$\frac{T}{V} = C$

Thus, P = constant gas absorbs heat $(Q) = nC_P\Delta T$

18.(D) (A) Immediately after impact, from conservation of momentum $mv = 2mv'_2$

$\therefore \quad v'_B = \frac{v}{2} \quad \ldots (1)$

m_B does not move immediately after impact, before the spring in relaxed.

Hence, velocity of centre of mass $v_{cm} = \frac{2m\left(\frac{v}{2}\right) + m(0)}{3m} = \frac{v}{3}$

(B) After impact we can use law of conservation of energy.

$\therefore \quad \frac{1}{2}(2m)\times\left(\frac{1}{2}v\right)^2 = \frac{1}{2}(2m)v_2^2 + \frac{1}{2}mv_3^2 + \frac{k}{2}(x_2 - x_3)^2 \quad \ldots (2)$

where v_2 and v_3 are velocities of $(m_1 + m_2)$ and m_3 resepctively and $(x_2 - x_3)$ is compression of the spring.

Using conservation of momentum, $\quad 2mv_2 + mv_3 = (2m)\frac{v}{2} \quad \therefore$

$v_2 = \frac{1}{2}(v - v_3)$

Total kinetic energy will be maximum when potential energy is minimum, i.e., zero. This occurs when $x_2 = x_3$.

Substituting the value of v_2 and setting $x_2 = x_3$ in equaiton (2), we get

$$\frac{1}{4}(v-v_3)^2+\frac{1}{2}v_3^2=\frac{v^2}{4}$$

i.e., $v_3(3v_3-2v)=0$

Hence $v_3=0$ or $v_3=\frac{2}{3}v$.

The maximum kinetic energy of individual particles occurs when total K.E. is maximum. Maximum K.E. of m_3 occurs, when $v_3=\frac{2v}{3}$

$$\therefore \qquad (K.E._{\max})=\frac{1}{2}m\left(\frac{2v}{3}\right)^2=\frac{2}{9}mv^2$$

where v_3 is maximum, v_2 is minimum. Hence, from law of conservation of momentum,

$$(2m)\frac{v}{2}=2m(v_2)_{\min}+\frac{2}{3}mv \quad \therefore \quad (v_2)_{\min}=\frac{v}{6}$$

(C) $\therefore$ When $v_2=\frac{v}{6}$; $v_3=\frac{2v}{3}$

(D) P.E. is maximum when compression in the spring is maximum. This occurs when the relative velocity of the two ends of the spring is zero. i.e., when (m_1+m_2) and m_3 move with common velocity. At this time

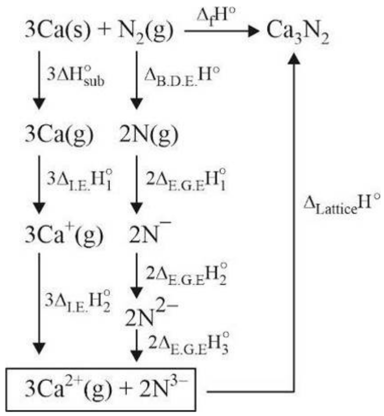
$$v_2=v_3=\frac{v}{3} \text{ (from conservation of momentum)}$$

$\therefore$ when $v_3=\frac{v}{3}$, P.E. is maximum.

CHEMISTRY

NUMERIC TYPE

1.(625) Born Haber's cycle for Ca_3N_2



$$\Delta_f H^\circ = 3\Delta_{sub}H^\circ + 3\Delta_{I.E.}H_1^\circ + 3\Delta_{I.E.}H_2^\circ + \Delta_{B.D.E}H^\circ + 2\Delta_{E.G.E}H_1^\circ + 2\Delta_{E.G.E}H_2^\circ + 2\Delta_{E.G.E}H_3^\circ + \left(-\Delta_{Lattice}H^\circ\right)$$

$\Delta_{Lattice}H^\circ$. = 625 kJ/mol

2.(756) $SO_4^{2-} \xrightarrow{2e^-} SO_3^{2-}$ $\qquad E_1 = -0.936$

$SO_3^{2-} \xrightarrow{2e^-} \frac{1}{2}S_2O_3^{2-}$ $\qquad E_2 = -0.576$

$SO_4^{2-} \xrightarrow{4e^-} \frac{1}{2}S_2O_3^{2-}$ $\qquad E^\circ = ?$

$-4 \times E^\circ = -2 \times -0.936 + (-2 \times -0.576)$

$\therefore \; E^\circ = -0.756$ Volt

3.(11.22) Let m.moles of BOH = x

$$BOH + HCl \longrightarrow BCl + H_2O$$

x	x/4	-	-
3x/4	-	x/4	

$$pOH = pK_b + \log\frac{x/4}{3x/4}$$

$$14 - pH = pK_b + \log\left(\frac{1}{3}\right)$$

$$14 - 9.24 = pK_b - \log 3$$

$$pK_b = 14 - 9.24 + \log 3$$

$$= 14 - 9.24 + 0.48$$

$$pK_b = 5.24$$

$BCl + NaOH \longrightarrow BOH + NaCl$

x/4 6 - -

Now, $\frac{x}{4} = 6$

$x = 24$

$[BOH] = \frac{24}{50} = 0.48$

$BOH(aq) \rightleftharpoons B^+(aq) + {}^-OH(aq)$

0.48 - -

0.48 – y y y

$\Rightarrow \frac{y^2}{0.48 - y} = K_b$

$\Rightarrow \frac{[HO^-]^2}{0.48} = K_b$

$\Rightarrow 2pOH = pK_b - \log\left(\frac{48}{100}\right)$

$\Rightarrow 2pOH = pK_b - \log 48 + \log 100$

$\Rightarrow 2pOH = 5.24 - \log(16 \times 3) + 2$

$\Rightarrow 2pOH = 5.24 - \log 2^4 - \log 3 + 2$

$\Rightarrow 2pOH = 5.24 - 1.2 - 0.48 + 2$

$\Rightarrow pOH = 2.78$

$\therefore pH = 11.22$

4.(4) $\underset{\text{Pyrolusite}}{MnO_2} + KOH + O_2 \longrightarrow \underset{\text{Dark green}}{\underset{(A)}{K_2MnO_4}} + H_2O$

$\underset{(A)}{K_2MnO_4} + H_2SO_4 \longrightarrow \underset{\text{Purple}}{\underset{(B)}{KMnO_4}} + MnO_2 + K_2SO_4 + H_2O$

$\text{alk. } \underset{(B)}{KMnO_4} + KI \longrightarrow \underset{(C)}{KIO_3} + MnO_2 + KOH$

$\underset{(B)}{KMnO_4} + H_2SO_4 + FeSO_4 \longrightarrow K_2SO_4 + MnSO_4 + Fe_2(SO_4)_3 + 8H_2O$

$KMnO_4 + H_2SO_4 \rightarrow \overset{(D)}{Mn_2O_7} + K_2SO_4 + 8H_2O$

$\downarrow \Delta$

$\underset{(E)}{MnO_2 + O_2}$

5.(32) $P_4 + 8SOCl_2 \longrightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

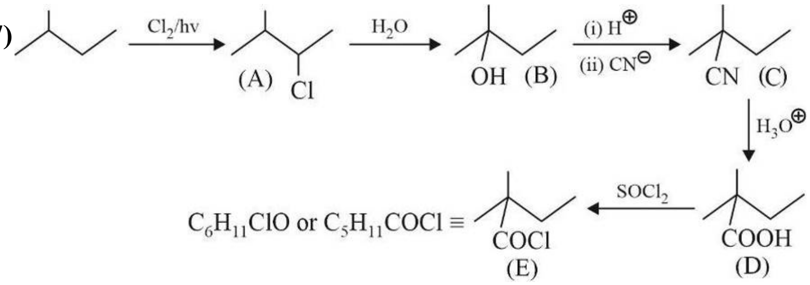
$5SO_2 + 2KMnO_4 + 2H_2O \longrightarrow 5SO_4^{2-} + 2Mn^{2+} + 2K^+ + 4H^+$

P_4 initial $= \frac{2.48}{124} \times 1000 = 20$ mmoles

SO_2 produced = 80 mmloes

$KMnO_4$ reacted $= \frac{2}{5} \times 80 = 32$ mmoles

6.(2.87)



Molar mass of E $= 6 \times 12 + 11 + 35.5 + 16 = 134.5$ gm / mol

Moles of E $= \frac{2.69}{134.5}$ mol $= 0.02$ mol

Moles of Cl = 0.02 mol

Moles of AgCl = 0.02 mol

Mass of AgCl $= 0.02 \times (108 + 35.5) = 2.87$ gm

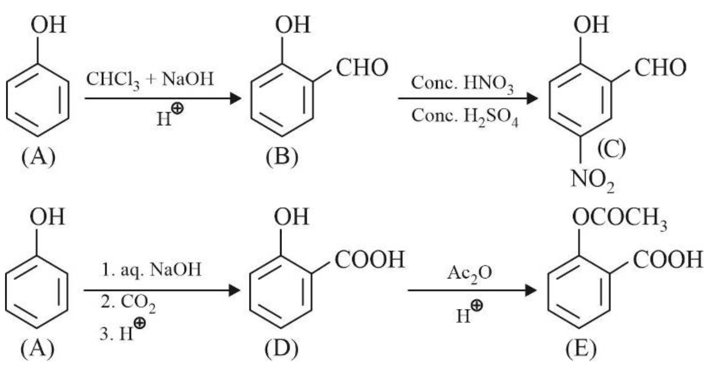
7.(4) (i) 3° carbocation is stable $\therefore$ S_N1 is possible

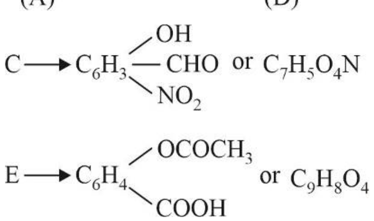
(iv) 2° carbocation fairly stable & undergoes rearrangement to form a stable 3° carbocation $\therefore$ S_N1 is possible

(v) Compound undergoes NGP in which 1st step is RDS hence it follows first order kinetics

(viii) Allyl carbocation is resonance stabilized $\therefore$ S_N1 is possible

8.(704)





$10 \text{ moles of } A \xrightarrow{70\%} 7 \text{ moles of } B \xrightarrow{80\%} 5.6 \text{ moles of } C$

$5.6 \text{ moles } C \longrightarrow 22.4 \text{ moles of oxygen}$

$10 \text{ moles of } A \xrightarrow{90\%} 9 \text{ moles of } D \xrightarrow{60\%} 5.4 \text{ moles of } E$

$5.4 \text{ moles } E \longrightarrow 21.6 \text{ moles of oxygen}$

So, total moles of oxygen = 22.4 + 21.6 = 44 moles

Total mass of oxygen $= \frac{44 \times 16}{1000} \text{kg} = 0.704 \text{kg} = 704 \text{gm}$

MULTIPLE CHOICE

9.(ABC) (A) Bond angle $OCl_2 > OF_2$ due to steric repulsion in OCl_2

(B) H_2O have greater boiling point than HF because in H_2O the extent of H-bonding is more than in HF

(C) $O(SiH_3)_2$ have more %S character than O in $O(CH_3)_2$ due to back bonding in $O(SiH_3)_2$

(D) $CHCl_3$ is more acidic than CHF_3 due to back bonding in $^{\ominus}CCl_3$

10.(AC) Macromolecular colloids are generally lyophilic in nature

Chemisorption is favoured by very high temperature and smaller surface area

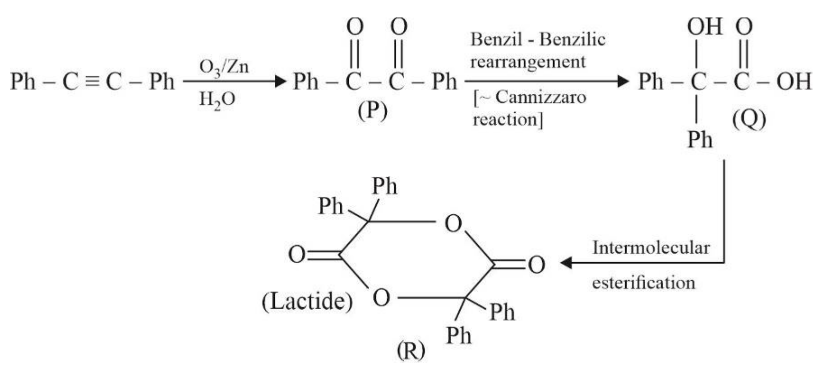
11.(ABCD) Fact based

12.(ABD) $Ni^{2+} + 2DMG \xrightarrow{NH_4OH} \underset{\text{Rose red ppt}}{[Ni(DMG)_2]} \downarrow$

$FeCl_3 + 3KSCN \longrightarrow Fe(SCN)_3 + 3KCl$

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 \uparrow + Cr_2O_3 + 4H_2O$

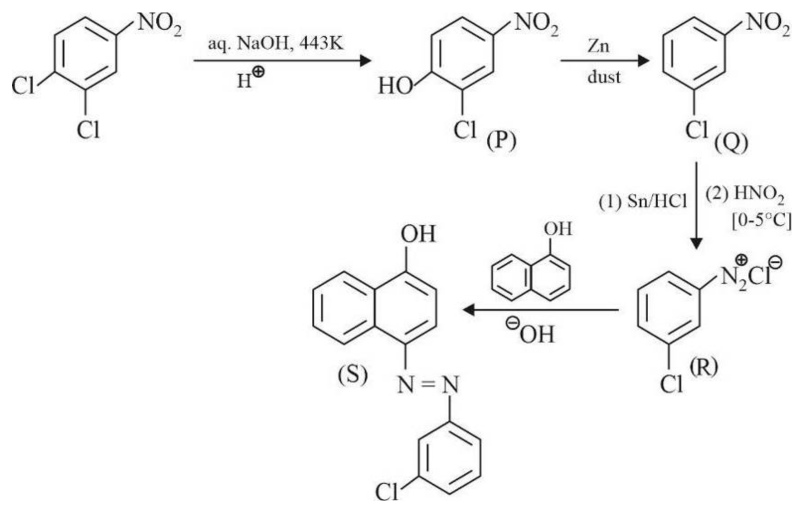
13.(AC)



Number of rings = 4 + 1 = 5

Number of ester linkages = 2

14.(BD)



MATRIX MATCH

15.(B) For 1st order reaction

$$[A] = [A_0]e^{-kt}$$

$$t_{1/2} = \frac{\ln 2}{k}$$

For zero order reaction

$$t_{1/2} = \frac{[A_0]}{2k}\ ;\ r = k$$

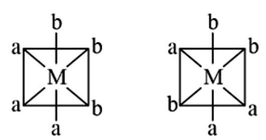
16.(C) $MSO_4 \xrightarrow{\Delta} MO + SO_2 + O_2$

$M = (Ba, Sr, Mg)$

$BaCrO_4$, $SrCrO_4$ forms yellow colour precipitate

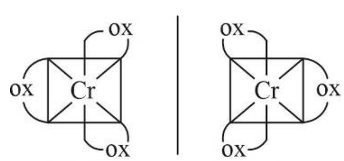
$BaSO_4$ and $SrSO_4$ are insoluble in water and forms white precipitate

17.(D) I. $[CoCl_3(NH_3)_3]$ is Ma_3b_3 type complexes show facial and meridional form



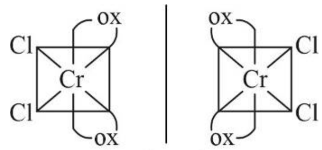
Facial Meridional

II. $[Cr(OX)_3]^{3-}$

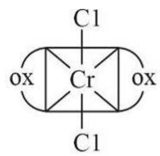


No. POS, optically active

III. $[CrCl_2(OX)_2]$

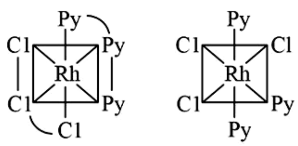


cis mirror image

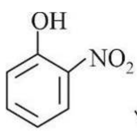


Trans has POS, optically inactive

IV. $[RhCl_3(Py)_3]$



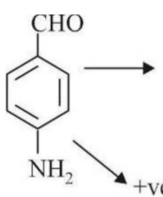
18.(D) I.



→ +ve Neutral $FeCl_3$ test (P)

→ It's sodium fusion extract gives prussian blue colour with $FeSO_4$ & H_2SO_4 due to presence of nitrogen (R)

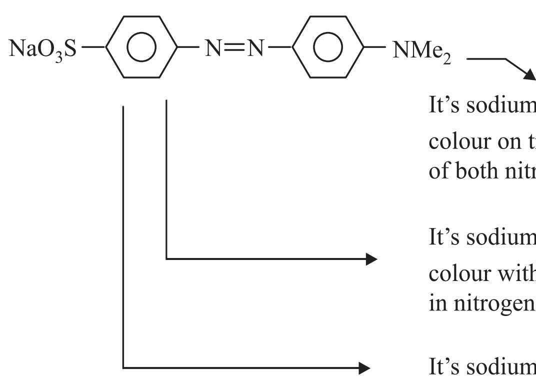
II.



→ It's sodium fusion extract gives prussian blue colour with $FeSO_4$ & H_2SO_4 due to presence of nitrogen (R)

→ +ve silver mirror test (Q)

III.



It's sodium fusion extract gives blood red colour on treatment with Fe^{+3} due to presence of both nitrogen & sulphur (S)

It's sodium fusion extract gives prussian blue colour with $FeSO_4$ & H_2SO_4 due to presence in nitrogen (R)

It's sodium fusion extract gives violet colour on treatment with sodium nitroprusside due to presence of sulphur (T)

IV. X : HCOOH $\longrightarrow$ gives +ve silver mirror test (Q)

Mathematics

NUMERIC TYPE

1.(2) Here, $y = \lim_{n\to\infty} y_n = \lim_{n\to\infty} \sum_{m=1}^{n} \tan^{-1}\left(\frac{1}{2m^2}\right)$

$$= \lim_{n\to\infty} \sum_{m=1}^{n} \tan^{-1}\left(\frac{2}{1+(2m+1)(2m-1)}\right) = \lim_{n\to\infty} \sum_{m=1}^{n} \tan^{-1}\left[\frac{(2m+1)-(2m-1)}{1+(2m+1)(2m-1)}\right]$$

$$= \lim_{n\to\infty} \sum_{m=1}^{n} (\tan^{-1}(2m+1) - \tan^{-1}(2m-1))$$

$$= \lim_{n\to\infty} (\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 5 - \tan^{-1} 3) + ... + (\tan^{-1}(2n+1) - \tan^{-1}(2n-1))$$

$$= \lim_{n\to\infty} (\tan^{-1}(2n+1) - \tan^{-1}(1)) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$\because\ B \to \left(1, \frac{\pi}{4}\right)$, i.e., coordinates of B approaches towards those of A.

Chord AB approaches to the tangent to $y = f(x)$ at A

$\therefore$ Slope of $AB = \left(\frac{d}{dx} \tan^{-1} x\right)_{at\, x=1}$

$= \left(\frac{1}{1+x^2}\right)_{at\, x=1} = \frac{1}{2} \quad \Rightarrow \quad (\text{Slope of } AB)^{-1} = 2$

2.(2) Here, $\lim_{x\to 0} \frac{\log_e\left[\cot\left(\frac{\pi}{4} - K_1 x\right)\right]}{\tan K_2 x} = 1$

$$\Rightarrow \lim_{x\to 0} \frac{\log\left[\cot\left(\frac{\pi}{4} - K_1 x\right) - 1 + 1\right]}{\tan K_2 x} = 1 \quad \Rightarrow \lim_{x\to 0} \frac{\log\left[1 + \frac{2\tan K_1 x}{1 - \tan K_1 x}\right]}{\tan K_2 x} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{\log\left(1 + \frac{2\tan K_1 x}{1 - \tan K_1 x}\right)}{\frac{2\tan K_1 x}{1 - \tan K_1 x}} \cdot \frac{\frac{2\tan K_1 x}{1 - \tan K_1 x}}{\tan K_2 x} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{2\tan K_1 x}{\tan K_2 x} = 1 \Rightarrow \lim_{x\to 0} \frac{\frac{2\tan K_1 x}{K_1 x} \cdot K_1 x}{\frac{\tan K_2 x}{K_2 x} \cdot K_2 x} = 1$$

$$\Rightarrow \frac{2K_1}{K_2} = 1 \Rightarrow 2K_1 = K_2$$

3.(8) Let the orthocenter be $O(x, y)$.

Side BC is perpendicular bisector of OE

$\therefore \; OD = DE$

$\Rightarrow \; \sqrt{(x-2)^2+(y-3)^2} = \sqrt{(5-2)^2+(5-3)^2}$

$\Rightarrow \; (x-2)^2+(y-3)^2 = (5-2)^2+(5-3)^2$

$\Rightarrow \; x^2+y^2-4x-6y=0 \;\Rightarrow\; x = 2 \pm \sqrt{13-(y-3)^2}$

$\Rightarrow \; y$ can take the values as $1, 2, 3, 4, 5, 6$

$\therefore$ Required probability $= \dfrac{6}{10} = \dfrac{3}{5} = \dfrac{m}{n}$ (given)

$\Rightarrow \; m = 3$ and $n = 5$ $\quad \therefore \; m+n=8$

4.(5) We have, $z_1(z_1^2-3z_2^2)=2$ ...(i)

$z_2(3z_1^2-z_2^2)=11$ ...(ii)

Multiplying equation (ii) by $i\left(\sqrt{-1}\right)$ and then adding in equation (i), we get,

$z_1^3-3z_1z_2^2+i(3z_1^2z_2-z_2^3)=2+11i$

$\Rightarrow \; (z_1+iz_2)^3 = 2+11i$ ...(iii)

Again, multiplying equation (ii) by $(-i)$ and then adding in equation (i) we get,

$z_1^3-3z_1z_2^2-i(3z_1^2z_2-z_2^3)=2-11i$

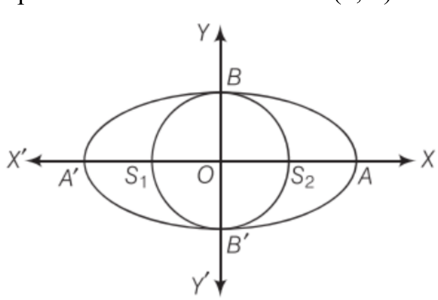
$\Rightarrow \; (z_1-iz_2)^3 = 2-11i$ ...(iv)

Now, on multiplying equation (iii) and (iv) we get,

$(z_1^2+z_2^2)^3 = 4+121 = 125 = 5^3 \quad \therefore \; z_1^2+z_2^2=5$

5.(2) Here, $|z-(4+8i)| = \sqrt{10}$

Represents a circle with centre (4, 8) and radius $\sqrt{10}$



Also, $|z-(3+5i)|+|z-(5+11i)| = 4\sqrt{5}$

Represents an ellipse

$\therefore \; |(3+5i)-(5+11i)| = \sqrt{4+36} = \sqrt{40} < 4\sqrt{5}$

With foci, $S_1(3,5)$ & $S_2(5,11)$

Distance between foci $= S_1S_2 = \sqrt{40} = 2\sqrt{10} =$ Diameter of circle

$2ae = 2\sqrt{10}$

$\Rightarrow \quad ae = \sqrt{10}$ & $2a = 4\sqrt{5} \Rightarrow \quad a = 2\sqrt{5}$

$\therefore \quad e = \dfrac{ae}{a} = \dfrac{1}{\sqrt{2}}$

Now, $b = a\sqrt{(1-e^2)} = 2\sqrt{5}\sqrt{\left(1-\dfrac{1}{2}\right)} = \sqrt{10} =$ Radius of circle

$\therefore$ Centre of the ellipse = Mid-point of S_1 & S_2

$= \dfrac{3+5i+5+11i}{2} = 4+8i$ i.e., (4, 8)

Which coincides with the centre of the circle and length of minor-axis is equal to the radius of the circle. Hence, there are only (2) two solutions of the given equations.

6.(2) $A_{n+1} = \dfrac{3(1+A_n)}{(3+A_n)}$. for $n=1$, $A_2 = \dfrac{3(1+A_1)}{(3+A_1)}$

For $n=2$, $A_3 = \dfrac{3(1+A_2)}{(3+A_2)} = \dfrac{3\left(1+\dfrac{3(1+A_1)}{(3+A_1)}\right)}{3+\dfrac{3(1+A_1)}{(3+A_1)}} = \dfrac{6+4A_1}{4+2A_1} = \dfrac{3+2A_1}{2+A_1}$

$\because$ Given sequence can be written as

$A_1, \dfrac{3(1+A_1)}{(3+A_1)}, \dfrac{(3+2A_1)}{(2+A_1)}, \ldots.$

Given, $A_1 > 0$ and sequence is decreasing, then

$A_1 > \dfrac{3(1+A_1)}{(3+A_1)}, \dfrac{3(1+A_1)}{(3+A_1)} > \dfrac{(3+2A_1)}{(2+A_1)}$

$\Rightarrow \quad A_1^2 > 3$ *or* $A_1 > \sqrt{3}$

$\therefore \quad A_1 = 2$ [least integral value of A_1]

7.(8) Since, $x \geq 1$, then $y \geq 2$ [$\because x < y$]

If $y = n$ then x takes values form 1 to $n-1$ and z can take the values from 0 to $n-1$ (i.e., n values)

Thus, for each values of $y(2 \leq y \leq 9), x$ & z take $n(n-1)$ values.

Hence, the 3-digit numbers are of the form xyz

$= \sum_{n=2}^{9} n(n-1) = \sum_{n=1}^{9} n(n-1)$ [$\because n=1, n(n-)=0$]

$= \sum_{n=1}^{9} n^2 - \sum_{n=1}^{9} n = \dfrac{9(9+1)(18+1)}{6} - \dfrac{9(9+1)}{2} = 285 - 45 = 240 = \lambda \qquad \therefore \dfrac{\lambda}{30} = 8$

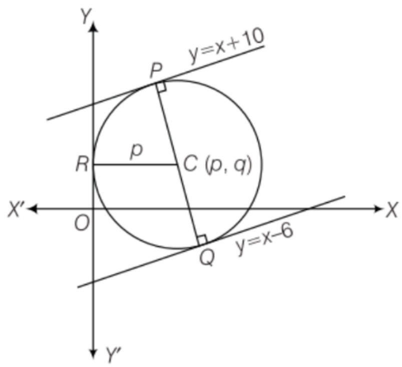
8.(6) $\because \quad CP = CR$

$$\Rightarrow \quad \frac{|p-q+10|}{\sqrt{2}} = p$$

$$p - q + 10 = p\sqrt{2} \qquad \ldots(i)$$

$$CP = CQ$$

$$\frac{p-q+10}{\sqrt{2}} = -\left(\frac{p-q-6}{\sqrt{2}}\right) \text{ or } p - q = -2 \qquad \ldots(ii)$$



From equation (i) and (ii), we get

$$p = 4\sqrt{2} \;\&\; q = 4\sqrt{2} + 2$$

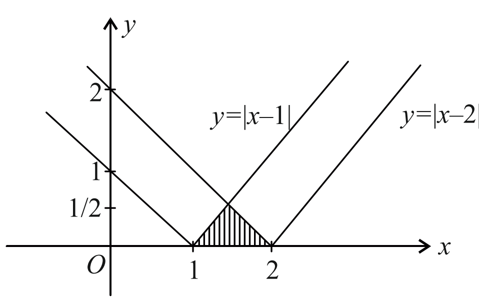
$$p + q = 2 + 8\sqrt{2} = a + b\sqrt{2}$$

$$\therefore \quad a = 2,\ b = 8$$

Hence, $|a - b| = |2 - 8| = 6$

MULTIPLE CHOICE

9.(BC)s Here, $\min\{|x-n|, |x-(n+1)|\}$ can be shown as



$$\therefore \; A_1 = \int_n^{n+1} (\min\{|x-n|, |x-(n+1)|\})\, dx$$

$$= \frac{1}{2} \times 1 \times \frac{1}{2} = \frac{1}{4}$$

$$A_2 = \int_{n+1}^{n+2} (|x-n| - |x-(n+1)|)\, dx$$

$= \int_1^2 (|t| - |t-1|)\,dt$ [put $x = n + t \Rightarrow dx = dt$]

$\int_1^2 (t - (t-1))\,dt = \int_1^2 1dt = (t)_1^2 = 1$

$A_3 = \int_{n+2}^{n+3} (|x - (n+4)| - |x - (n+3)|)\,dx$

$= \int_2^3 (|t-4| - |t-3|)\,dt$ [put $x = n + t \Rightarrow dx = dt$]

$= \int_2^3 ((4-t) - (3-t))\,dt = \int_2^3 1dt = 1$

Also, $g(x) = A_1 + A_2 + A_3 = \frac{1}{4} + 1 + 1 = \frac{9}{4}$

$\therefore \sum_{n=1}^{100} g(x) = g(1) + g(2) + g(3) + ... + g(100)$

$= \frac{9}{4} + \frac{9}{4} + ... + \frac{9}{4} = \frac{900}{4}$

10.(ABC)

$\sin^2 x - a\sin x + b = 0$ has only one solution in $(0, \pi)$

$\Rightarrow$ $\sin x = 1$ gives one solution and $\sin x = \alpha$ gives other solution such that $\alpha > 1$ or $\alpha \le 0$

$\Rightarrow$ $(\sin x - 1)(\sin x - \alpha)$ is the same equation as $\sin^2 x - a\sin x + b = 0$

$\Rightarrow$ $1 + \alpha = a$ and $\alpha = b$

$\Rightarrow$ $1 + b = a$ and $b > 1$ or $b \le 0$

$\Rightarrow$ $b \in (-\infty, 0] \cup [1, \infty)$ and $a \in (-\infty, 1] \cup [2, \infty)$.

11.(ABC) Equation of line AB is $\frac{x}{2} = \frac{y}{-3} = \frac{z}{6}$

Its DR's are $< 2, -3, 6 >$

Let the coordinates be $< 2r, -3r, 6r >$

DR's of PN are $< 2r - 1, -3r - 2, 6r - 5 >$

It is perpendicular to AB

$\therefore \quad 2(2r-1) - 3(-3r-2) + 6(6r-5) = 0$

$4r - 2 + 9r + 6 + 36r - 30 = 0$

$49r = 26$ i.e. $r = \frac{26}{49}$

(A) $\therefore$ Coordinates of N are $\left(\frac{52}{49}, -\frac{78}{49}, \frac{156}{49}\right)$

(B) Let the coordinates of Q be $(2r, -3r, 6r)$, then DR's of PQ are $< 2r-1,\ -3r-2,\ 6r-5 >$
Since PQ is parallel to the plane

$\therefore\ 3(2r-1)+4(-3r-2)+5(6r-5)=0$

$6r-3-12r-8+30r-25=0$

$24r=36,\ r=\frac{3}{2}$

$\therefore$ Coordinates of Q are $\left(3, -\frac{9}{2}, 9\right)$

Equation of PN is $\frac{x-1}{3}=\frac{y-2}{-176}=\frac{z-5}{-89}$

12.(ABC) Equation of any plane passing through $(-2,-2,2)$ is

$A(x+2)+B(y+2)+C(z-2)=0$

Since it contains the line joining $(1,1,1)$ and $(1,-1,2)$ these points also lie on this plane

$\Rightarrow\ 3A+3B-C=0\ \&\ 3A+B+0=0$

$\Rightarrow\ \frac{A}{1}=\frac{B}{-3}=\frac{C}{-6}$

So, the equation of the plane is

$(x+2)-3(y+2)-6(z-2)=0$

$x-3y-6z+8=0$

$\frac{x}{-8}+\frac{y}{\frac{8}{3}}+\frac{z}{\frac{8}{6}}=1\ \Rightarrow\ a=8,\ b=\frac{8}{3},\ c=\frac{8}{6}$

$a=3b,\ b=2c,\ a+b+c=12$

$a+2b+2c=16$

13.(AB) Given equation of the ellipse is $4x^2+9y^2=36$

$\frac{x^2}{9}+\frac{y^2}{4}=1$ ...(i)

The equation of tangent at $(3\cos\theta,\ 2\sin\theta)$ is

$\frac{x}{3}\cos\theta+\frac{y}{2}\sin\theta=1$

Which meets the tangent at $x=3$ and $x=-3$ at the extremities of major axis

$T\equiv\left(3, \frac{2(1-\cos\theta)}{\sin\theta}\right)$

$T'\equiv\left(-3, \frac{2(1+\cos\theta)}{\sin\theta}\right)$

$\therefore$ Equation of circle on TT' as diameter is

$$(x-3)(x+3)+\left(y-\frac{2(1-\cos\theta)}{\sin\theta}\right)\left(y-\frac{2(1+\cos\theta}{\sin\theta}\right)=0$$

$$\Rightarrow \quad x^2+y^2-\frac{4}{\sin\theta}\cdot y-5=0$$

$$(x^2+y^2-5)-\frac{4}{\sin\theta}y=0 \qquad \ldots\text{(ii)}$$

Clearly equation (ii) passes through point of intersection of

$x^2+y^2-5=0$ and $y=0$ i.e., $\left(\pm\sqrt{5},0\right)$

14.(ACD) Applying $C_2 \to C_2 - C_1 - 2C_3$, then

$$\begin{vmatrix} a^2 & -(b^2+c^2) & bc \\ b^2 & -(c^2+a^2) & ca \\ c^2 & -(a^2+b^2) & ab \end{vmatrix} = -\begin{vmatrix} a^2 & b^2+c^2 & bc \\ b^2 & c^2+a^2 & ca \\ c^2 & a^2+b^2 & ab \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$, then

$$= -\begin{vmatrix} a^2 & a^2+b^2+c^2 & bc \\ b^2 & a^2+b^2+c^2 & ca \\ c^2 & a^2+b^2+c^2 & ab \end{vmatrix}$$

Appling $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, then

$$= -\begin{vmatrix} a^2 & a^2+b^2+c^2 & bc \\ b^2-a^2 & 0 & c(a-b) \\ c^2-a^2 & 0 & -b(c-a) \end{vmatrix}$$

$$= (a^2+b^2+c^2)\begin{vmatrix} -(a+b)(a-b) & c(a-b) \\ (c+a)(c-a) & -b(c-a) \end{vmatrix}$$

$$= (a-b)(c-a)(a^2+b^2+c^2)\begin{vmatrix} -(a+b) & c \\ c+a & -b \end{vmatrix}$$

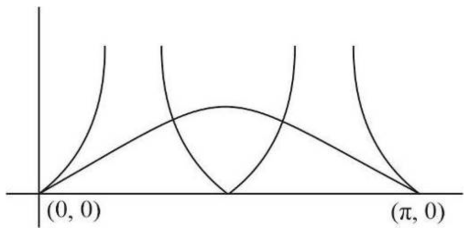
Applying $C_1 \to C_1 - C_2$, then

$$= (a-b)(c-b)(a^2+b^2+c^2)\begin{vmatrix} -(a+b+c) & c \\ (a+b+c) & -b \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

MATRIX MATCH

15.(B) (A) Clearly, number of solution of $|\tan 2x| = \sin x$ in $(0, \pi)$ are 2.



(B) $$\tan 4A = \frac{2\tan 2A}{1-\tan^2 2A} = \frac{\frac{4\tan A}{1-\tan^2 A}}{1-\left(\frac{2\tan A}{1-\tan^2 A}\right)^2}$$

$$\Rightarrow \quad \tan 4A = \frac{4\tan A(1-\tan^2 A)}{1+\tan^4 A - 6\tan^2 A}$$

$$= 4\tan A - 4\tan^3 A + (6\tan^2 A - \tan^4 A - 1)\tan 4A = 0$$

If $A = \frac{\pi}{16}$

$$\Rightarrow \quad 4\tan\frac{\pi}{16} - 4\tan^3\frac{\pi}{16} + 6\tan^2\frac{\pi}{16} - \tan^4\frac{\pi}{16} = 1$$

Required value is 2

(C) $$\tan(p\cot x) = \cot(p\tan x)$$

$$\tan(p\cot x) = \tan\left(\frac{\pi}{2} - p\tan x\right)$$

$$p\cot x = n\pi + \frac{\pi}{2} - p\tan x$$

$$p = \frac{n\pi + \frac{\pi}{2}}{\tan x + \cot x} = \frac{\pi}{2}\sin x\cos x \qquad (\because x \in [0, \pi])$$

$$P_{\max} = \frac{\pi}{4}; \frac{4P_{\max}}{\pi} = 1$$

(D) $$5^{\cos^2 2x + 2\sin^2 x} + 5^{2\cos^2 x + \sin^2 2x} = 126$$

$$5^{\cos^2 2x + 2\sin^2 x} + 5^{2-2\sin^2 x + 1 - \cos^2 2x} = 126$$

$$5^{\cos^2 2x + 2\sin^2 x} + 5^{3-(\cos^2 2x + 2\sin^2 x)} = 126$$

Put $\cos^2 2x + 2\sin^2 x = y$, we get

$$5^y + 5^{3-y} = 126$$

$$5^y + \frac{125}{5^y} = 126 \Rightarrow 5^y = 125, 1 \Rightarrow y = 3, 0$$

$$\cos^2 2x + 2\sin^2 x = 3 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos^2 2x + 2\sin^2 x \neq 0 \; ; \frac{2x}{\pi} = 1, 3$$

16.(A) A and B are independent events.

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{12}$$

$$P(A \cap \bar{B}) = P(A) \cdot P(\bar{B}) = \frac{1}{3} \times \left(1 - \frac{1}{4}\right) = \frac{1}{4}$$

$$P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B) = \left(1 - \frac{1}{3}\right) \cdot \frac{1}{4} = \frac{1}{6}$$

(A) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{1}{3} = \lambda_1$

$\therefore\ 12\lambda_1 = 4$ [natural number and composite number]

(B) $P\left(\frac{A}{A \cup B}\right) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$

$$= \frac{P(A)}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)} - P(A \cap B)$$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{4} - \frac{1}{12}} = \frac{2}{3} = \lambda_2$$

$\therefore\ 9\lambda_2 = 6$

[natural number, composite number and perfect number]

(C) $P(A \cap \bar{B}) \cup (\bar{A} \cap B)) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$$= \frac{1}{4} + \frac{1}{6} = \frac{5}{12} = \lambda_3$$

$\therefore\ 12\lambda_3 = 5$ [prime number and natural number]

(D) $P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B)$

$$= \left(1 - \frac{1}{3}\right) + \frac{1}{4} - \frac{1}{6} = \frac{3}{4} = \lambda_4$$

$\therefore\ 12\lambda_4 = 9$ [natural number and composite number]

17.(C) (A) $f(x)=\begin{vmatrix} x^2 & 2x & 1+x^2 \\ x^2+1 & x+1 & 1 \\ x & -1 & x-1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_3$, then

$$f(x)=\begin{vmatrix} -1 & 2x & 1+x^2 \\ x^2 & x+1 & 1 \\ 1 & -1 & x-1 \end{vmatrix}$$

Expanding along R_1, then

$f(x) = -(x^2-1+1) - 2x(x^3-x^2-1) + (1+x^2)(-x^2-x-1)$

$= -3x^4 + x^3 - 3x^2 + x - 1$ ...(i)

According to the question, we get

$f(x) = a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$ ...(ii)

From equation (i) and (ii), we get

$a_0 = -3,\ a_1 = 1,\ a_2 = -3,\ a_3 = 1,\ a_4 = -1$

(A) $a_0^2 + a_1 = (-3)^2 + 1 = 9 + 1 = 10 = 2\times 5$

(B) $a_2^2 + a_4 = (-3)^2 - 1 = 9 - 1 = 8 = 2\times 4$

(C) $a_0^2 + a_2 = (-3)^2 - 3 = 9 - 3 = 6 = 2\times 3$

(D) $a_4^2 + a_3^2 + a_1^2 = (-1)^2 + (1)^2 + (1)^2 = 1+1+1 = 3$

18.(D) (A) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = f(2)$

$$\Rightarrow \lim_{h\to 0} a^{2-h} = \lim_{h\to 0} \frac{b((2+h)^2-b^2)}{2+h-2} = 8$$

$$\Rightarrow a^2 = b\lim_{h\to 0}\frac{(2+h)^2-b^2}{h} = 8$$

At $h\to 0,\ (2+h)^2 - b^2 \to 0$

$\therefore\ b^2 = 4\ \&\ a^2 = 8$

$\therefore$ Locus of the pair of perpendicular tangents to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$\therefore$ required locus is

$x^2 + y^2 = a^2 + b^2 = 8 + 4 = 12$

$\Rightarrow\ r^2 = 12$

(B) Major axis on the line $y = 2$ and minor axis on the line $x = -1$

Centre of ellipse is $(-1, 2)$

$\Rightarrow \ h = -1, k = 2$

Also, $2a = 10$ and $2b = 4$

$M = a^2 = 25$

$N = b^2 = 4$

Now, $h + k + M + N = -1 + 2 + 25 + 4 = 30$

(C) $a = 5, b = 4$

$\because \ b^2 = a^2(1 - e^2) \Rightarrow 16 = 25(1 - e^2)$

$\therefore \ e = \frac{3}{5}$

Foci $(\pm 3, 0)$

$SA = 2$ [A and A' are vertices]

Also gives PS = 2

$\therefore$ P coincides with A and Q coincides with A'

$\therefore PQ = 2a = 10$

(D) Let $\left(\sqrt{27}\cos\theta, \sqrt{48}\sin\theta\right)$ be a point on the ellipse $\frac{x^2}{27} + \frac{y^2}{48} = 1$

Equation of tangent at $\left(\sqrt{27}\cos\theta, \sqrt{48}\sin\theta\right)$ is

$$\frac{x\cos\theta}{\sqrt{27}} + \frac{y\sin\theta}{\sqrt{48}} = 1$$

Slope $= -\frac{\sqrt{48}}{\sqrt{27}} \cdot \frac{\cos\theta}{\sin\theta} = -\frac{4}{3}$

$\tan\theta = 1$

$\Rightarrow \ \theta = \frac{\pi}{4}$

Equation of tangent is $\frac{x}{\sqrt{54}} + \frac{y}{\sqrt{96}} = 1$

Area of triangle $= \frac{1}{2} \times 3\sqrt{6} \times 4\sqrt{6} = 36$